

# COMPLEX SYSTEMS MANIFESTATIONS OF THE ONSET OF CHAOS VIA INTERMITTENCY



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# Transitions to chaos in complex systems...

How much was known, fifteen years ago?

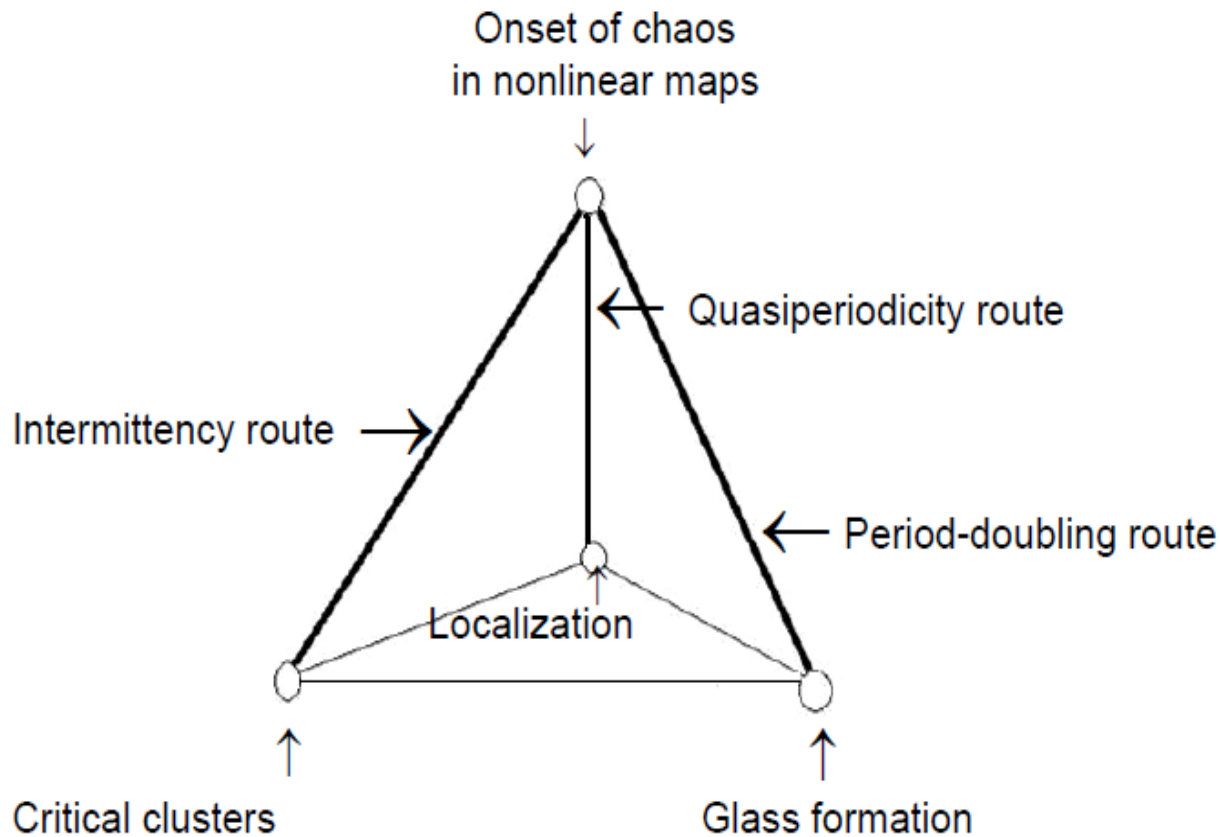
Which are the relevant recent advances? Is the dynamics *fully understood now*?

# PROJECT SUMMARY

- Motivated by answering these questions, Dr. Robledo and his team developed two types of tools for research: the dynamics IN and the dynamics TOWARDS the attractors that represent transitions to chaos.
- These are combined or complemented with other methodologies that correspond to stochastic processes. For example, in the physics of condensed matter there are results related to critical fluctuations, vitreous dynamics and location transition

# PROJECT DETAILS

## Description



# RESEARCH QUESTIONS

Problem :

A remarkable equivalence between the dynamics of an intermittent nonlinear map and the wave transport. This strict analogy shows in detail the nature of the mobility, **but it is not studied, not at the localized-to-extended transition in systems of scatterers.**

# PROPOSAL DEVELOPMENT

- ◉ We propose models of collective behavior active particles or agents, representing complex systems.

Complex systems manifestations of the onset of chaos via intermittency

- ◉ The pertinence of this work lies in the importance of the various applications of active particle models in different branches of knowledge.

# BACKGROUND: THEORETICAL FRAMEWORK AND METHODS

- ◉ One of the lines of research in more robust complex systems is based on an important result: the transition to chaos via intermittency, which expresses a universal behavior shared by a number of complex systems, and which has been documented phenomenologically over the years , under the names of the laws of Benford and Zipf.
- ◉ This was demonstrated, from a simple hypothesis, centered on the occupation of configurations in collectivities of numerical data, that these laws obey a statistical mechanical structure and thermodynamics. Other works established that the relationship between the accumulation states of thermodynamic systems and the transition to chaos via the phenomenon of intermittence, satisfy all the requirements of a hierarchical system composed of modules at different levels whose behavior shows separation in time scales and properties emerging from complex systems.

**Note:** Can review the theory in the references

# WHAT DO WE KNOW?

Low-dimensional nonlinear map

Transitions to chaos: Tangent bifurcation & intermittency

- ◉ Anomalous dynamics at the intermittency onset of chaos

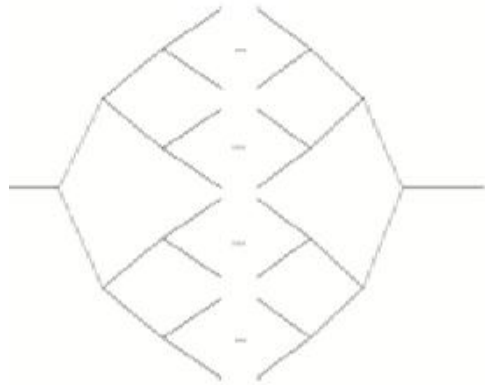
Features at transition between order and chaos

- ◉ Vanishing Lyapunov exponent coefficient
- ◉ Fluctuations that grow indefinitely

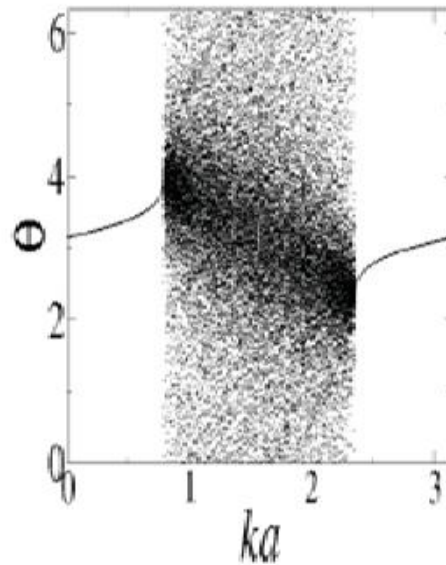


# ROUTE TO CHAOS: INTERMITTENCY

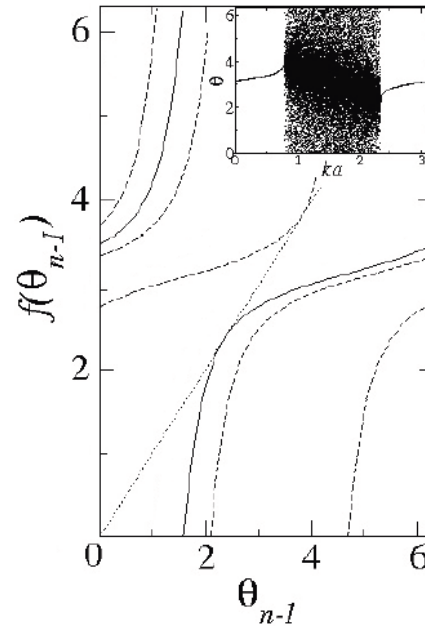
## Localization transition



Double Cayley tree



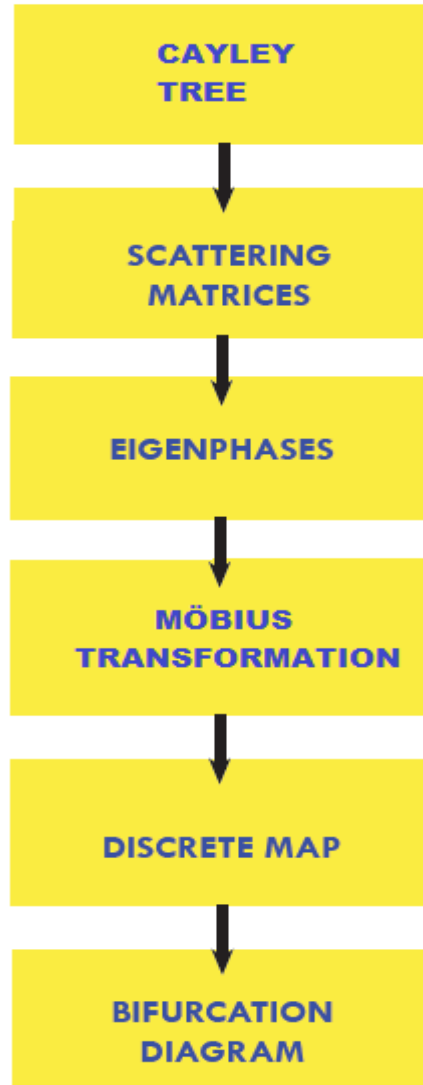
**Figure.** The double Cayley tree (from Ref. [3]) that can be used to describe the electronic transport.



**Figure.** Periodic and chaotic attractors (from Ref. [3]) obtained in the study of the equivalence between electronic transport and an intermittent nonlinear map.

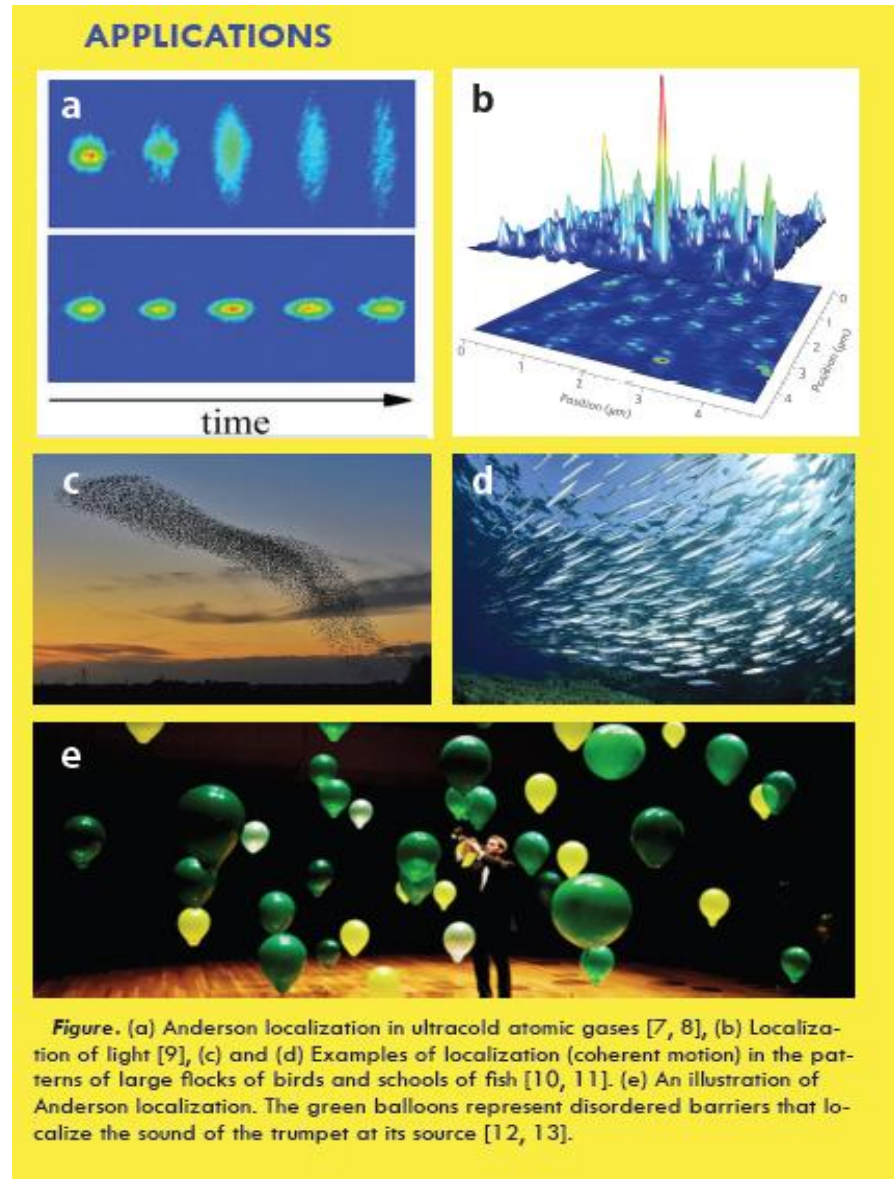
M. Martínez-Mares, A. Robledo, Equivalence between the mobility edge of electronic transport on disorderless networks and the onset of chaos via intermittency in deterministic maps. doi:10.1103/PhysRevE.80.045201.

# METHODS



# EXPECTED OUTCOMES

- Quantum chaos (critical fluctuations)
- Biological phenomena (localization)



# REFERENCES

- [1] G.C. Yalcin, A. Robledo, M. Gell-Mann, Incidence of  $q$  statistics in rank distributions, *Proc. Natl. Acad. Sci.* 111 (2014) 14082–14087. doi:10.1073/pnas.1412093111.
- [2] G.C. Yalcin, C. Velarde, A. Robledo, Entropies for severely contracted configuration space, *Heliyon*. 1 (2015) e00045. doi:10.1016/j.heliyon.2015.e00045.
- [3] A. Robledo, L.G. Moyano,  $q$ -deformed statistical mechanical property in the dynamics of trajectories en route to the Feigenbaum attractor, *Phys. Rev. E*. 77 (2008) 36213. doi:10.1103/PhysRevE.77.036213.
- [4] J.A. Robledo,  $q$ -deformed statistical-mechanical structure in the dynamics of the Feigenbaum attractor, *J. Phys. Conf. Ser.* 246 (2010) 12025. doi:10.1088/1742-6596/246/1/012025.
- [5] L.G. Moyano, A. Robledo, Dynamics towards the Feigenbaum attractor, *J. Phys.* 39 (2009) 364–370.
- [6] J.A. Diaz-Ruelas, A. Robledo, Emergent statistical-mechanical structure in the dynamics along the period-doubling route to chaos, *EPL (Europhysics Lett.)* 105 (2014) 40004. doi:10.1209/0295-5075/105/40004.
- [7] J.A. Diaz-Ruelas, A. Robledo, An unorthodox thermal system analogue of the onset of chaos, *EPJ-ST.* 172 (2016).
- [8] B. Luque, F.J. Ballesteros, Á.M. Núñez, A. Robledo, Quasiperiodic graphs: structural design, scaling and entropic properties, (2012). doi:10.1007/s00332-012-9153-2.
- [9] B. Luque, M. Cordero-Gracia, M. Gómez, A. Robledo, Quasiperiodic graphs at the onset of chaos, *Phys. Rev. E*. 88 (2013) 62918. doi:10.1103/PhysRevE.88.062918.
- [10] H. Hernández-Saldaña, A. Robledo, Fluctuating dynamics at the quasiperiodic onset of chaos, Tsallis -statistics and Mori's  $\gamma$ -phase thermodynamics, *Phys. A Stat. Mech. Its Appl.* 370 (2006) 286–300. doi:10.1016/j.physa.2006.03.018.
- [11] C. Beck, F. Schlögl, *Thermodynamics of chaotic systems*, Cambridge University Press, Cambridge, 1993.
- [12] F. Baldovin, A. Robledo, Parallels between the dynamics at the noise-perturbed onset of chaos in logistic maps and the dynamics of glass formation, *Phys. Rev. E*. 72 (2005) 66213. doi:10.1103/PhysRevE.72.066213.
- [13] J.A. Robledo, Critical fluctuations, intermittent dynamics and Tsallis statistics, *Phys. A Stat. Mech. Its Appl.* 344 (2004) 631–636. doi:10.1016/j.physa.2004.06.043.
- [14] J.A. Robledo, Génesis de una nueva física estadística, Fondo de Cultura Económica, Ciudad de México, 2006.
- [15] E. Mayoral, A. Robledo, A Recent Appreciation of the Singular Dynamics at the Edge of Chaos, in: *Logist. Map Route to Chaos*, Springer-Verlag, Berlin/Heidelberg, n.d.: pp. 339–354. doi:10.1007/3-540-32023-7\_19.
- [16] J.M. Martínez-Mares, V. Domínguez-Rocha, A. Robledo, Typical length scales in conducting disorderless networks, *Eur. Phys. J. Spec. Top.* 226 (2017) 417–425. doi:10.1140/epjst/e2016-60129-x.
- [17] B. Luque, L. Lacasa, F.J. Ballesteros, A. Robledo, Feigenbaum Graphs: A Complex Network Perspective of Chaos, *PLoS One*. 6 (2011) e22411. doi:10.1371/journal.pone.0022411.
- [18] J.M. Riquelme-Galván, A. Robledo, Dual characterization of critical fluctuations: Density functional theory & nonlinear dynamics close to a tangent bifurcation, *Eur. Phys. J. Spec. Top.* 226 (2017) 433–442. doi:10.1140/epjst/e2016-60268-0.
- [19] D. Vilone, A. Robledo, A. Sánchez, Chaos and Unpredictability in Evolutionary Dynamics in Discrete Time, *Phys. Rev. Lett.* 107 (2011) 38101. doi:10.1103/PhysRevLett.107.038101.
- [20] F. Baldovin, A. Robledo, Universal renormalization-group dynamics at the onset of chaos in logistic maps and nonextensive statistical mechanics, *Phys. Rev. E*. 66 (2002) 45104. doi:10.1103/PhysRevE.66.045104.
- [21] J.A. Robledo, Intermittency at critical transitions and aging dynamics at the onset of chaos, *Pramana*. 64 (2005) 947–956. doi:10.1007/BF02704156.
- [22] J.A. Robledo, L.G. Moyano, Statistical Mechanics for the Feigenbaum attractor, *Complexity, Metastability and Nonextensivity.* (2007) 114–117.
- [23] E. Mayoral, A. Robledo, Multifractality and nonextensivity at the edge of chaos of unimodal maps, *Phys. A Stat. Mech. Its Appl.* 340 (2004) 219–226. doi:10.1016/j.physa.2004.04.010.
- [24] C. Beck, F. Schlogl, *Thermodynamics of Chaotic Systems*, Cambridge University Press: Cambridge, United Kingdom, 1993.
- [25] Y. Jiang, M. Martínez-Mares, E. Castaño, A. Robledo, Möbius transformations and electronic transport properties of large disorderless networks, *Phys. Rev. E*. 85 (2012) 57202. doi:10.1103/PhysRevE.85.057202.
- [26] J.M. Martínez-Mares, A. Robledo, Equivalence between the mobility edge of electronic transport on disorderless networks and the onset of chaos via intermittency in deterministic maps, *Phys. Rev. E*. 80 (2009) 45201. doi:10.1103/PhysRevE.80.045201.
- [27] J.A. Diaz-Ruelas, M.A. Fuentes, A. Robledo, Scaling of distributions of sums of positions for chaotic dynamics at band-splitting points, *EPL (Europhysics Lett.)* 108 (2014) 20008. doi:10.1209/0295-5075/108/20008.
- [28] J.C. Altamirano, A. Robledo, Generalized Thermodynamics Underlying the Laws of Zipf and Benford, in: *Lect. Notes Inst. Comput. Sci. Soc. Informatics Telecommun. Eng.*, Springer, London, 2009: pp. 2232–2237.